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Norton's Theorem: Graph Theoretic Approach

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ABSTRACT : Graph theory, a branch of Mathematics plays a vital role in studying interdisciplinary subjects such as physics, Chemistry, Engineering etc. Study of the properties of electrical circuits with the help of graph theory is a growing trend in mathematical and electrical fields. Electrical circuits consists of nodes and branches which obeys Kirchhoff's current laws, Kirchhoff's voltage laws etc. There are various well known theorems such as Norton's theorem, Thevenin's theorem, Superposition theorem, Millman Theorem etc for network analysis. In this paper, we try to analyze Norton's theorem with the help of Graph theorem.

KEYWORDS : Graph theory, Electrical circuits, Branch current, Loop current, 2000 A.M.S. subject classification: . 05C50, 11F27.

I. INTRODUCTION

The electrical circuits [3] are the main building blocks of electrical appliances. Without the electrical appliances today's civilization become fully paralyzed. Now-a-days Graph theory [9],[8], a branch of Mathematics has impact on almost every other branch. In engineering mathematics also, graph theory plays a vital rule. Study of the properties of electrical circuits [7] with the help of graph theory is a growing trend in mathematical and electrical fields. Electrical circuits consists of nodes [3] and branches [3] which obeys Kirchhoff's current laws, Kirchhoff's voltage law [7] etc. The scientists now-a-days engage themselves to construct simple cost effective circuits. There are various well known theorems such as Norton's theorem, Thevenin's theorem [3], Superposition theorem [7], Millman's Theorem [11],[3] etc to construct simple circuits from the complicated circuits. In the graph of the circuits, all the elements of the circuits are replaced by lines with circles or dots at both ends. The lines represent branches and the dots represent nodes of the circuits. From these graphs, we can find matrices related to them by different rules. In our work we try to study electrical networks [1][5][22][2], and its simplified circuits and their associated currents and voltages through network equilibrium[12][13][14][15] equations which obey the laws of electricity.

1.1 Graph: A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$. The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. So in a graph a vertex set is a set of points $\{x_1, x_2, x_3 \dots x_n\}$ and edge is a line which connects two points x_i and x_j . Such graphs are called undirected graphs. A directed graph is similar to an undirected graph except the edge set $E \in V \times V$.

1.2 ELECTRICAL CIRCUITS: An electrical circuit consists of internally connected elements viz resistors, capacitors, inductors, diodes, transistors etc. The behaviour of an electrical circuits generally depends upon two factors.

- a) The characteristic of each of internally connected elements
- b) The rule by which they are connected together.

The second factor gives a relation between electrical circuits with graph theory. A two terminal electrical element can be represented by an edge e_k . Associate with each edge there are two variables $V_k(t)$ and $i_k(t)$. The variable $V_k(t)$ is called the edge voltage and may be regarded as cross variable because it exist across the two end points. The other variable is called edge current and may be regarded as through variable because it flows through the edge. These variables must also obey the two laws of Kirchhoff's.

1.3 Kirchhoff's Current Law (KCL): For any lumped electrical network, at any time the net sum (taking into account the orientations) of all the currents leaving any node or vertex is zero. That is at r^{th} vertex of the corresponding digraph, we must have

$$\sum_{k=1}^e a_{rk} i_k(t) = 0$$

Where a_{rk} is the rk th entry of the incidence matrix A of G and $i_k(t)$ is the amount of current flowing through the k th edge of G .

1.4 Kirchhoff's Voltage Law (KVL) : For any lumped electrical network, at any time the net sum (taking into account the orientations) of the voltages around a loop (i.e. circuit) is zero. In terms of the corresponding digraph, for the r th circuit we must have

$$\sum_{k=1}^e b_{rk} v_k(t) = 0$$

Where b_{rk} is the rk th entry of the circuit matrix B of G and $V_k(t)$ is the amount of voltage across the k th edge.

1.5 From Circuit to Graph :

A graph can be obtained from a circuit. We identify the graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. The edge between i th and j th vertices can be denoted by $\{i, j\}$ ignoring the direction. Similarly the notation (i, j) can be used for oriented edges, where i is the start vertex and j is the end vertex. For example consider the circuit and its graph in the figure. There are five vertices and seven edges in the graph obtained from the given circuit. An edge is sometimes called branch and a vertex is called node in case of electrical circuits.

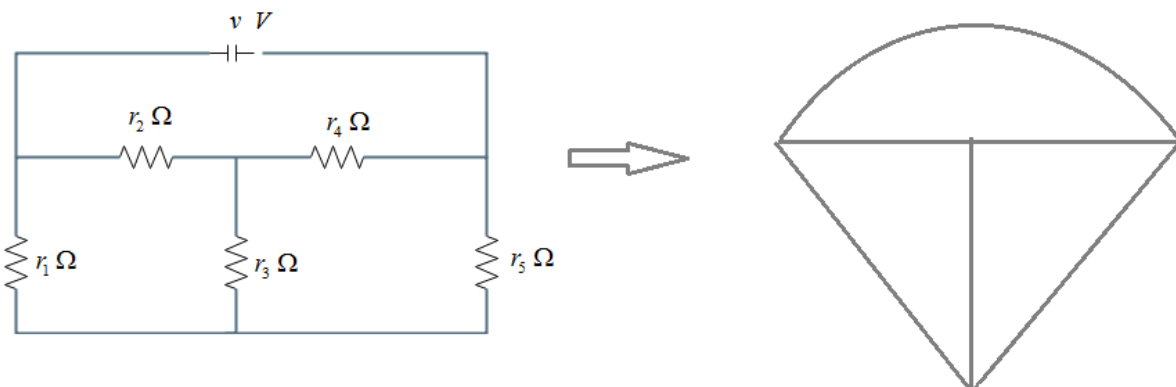


Figure:1. Circuit and its corresponding graph

1.6 MATRICES ASSOCIATED TO A GRAPH :

1.6.1 Fundamental Tie set matrix (Fundamental loop matrix) :

This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branch of trees). Here we assume that the direction of loop currents and direction of the link is same.

So in the matrix

- $b_{ij} = 1$, if the branch b_j in the fundamental loop i and their reference direction oriented same
- $b_{ij} = -1$ if the branch b_j in the fundamental loop i and their reference direction oriented opposite
- $b_{ij} = 0$ if the branch b_j is not in the fundamental loop i

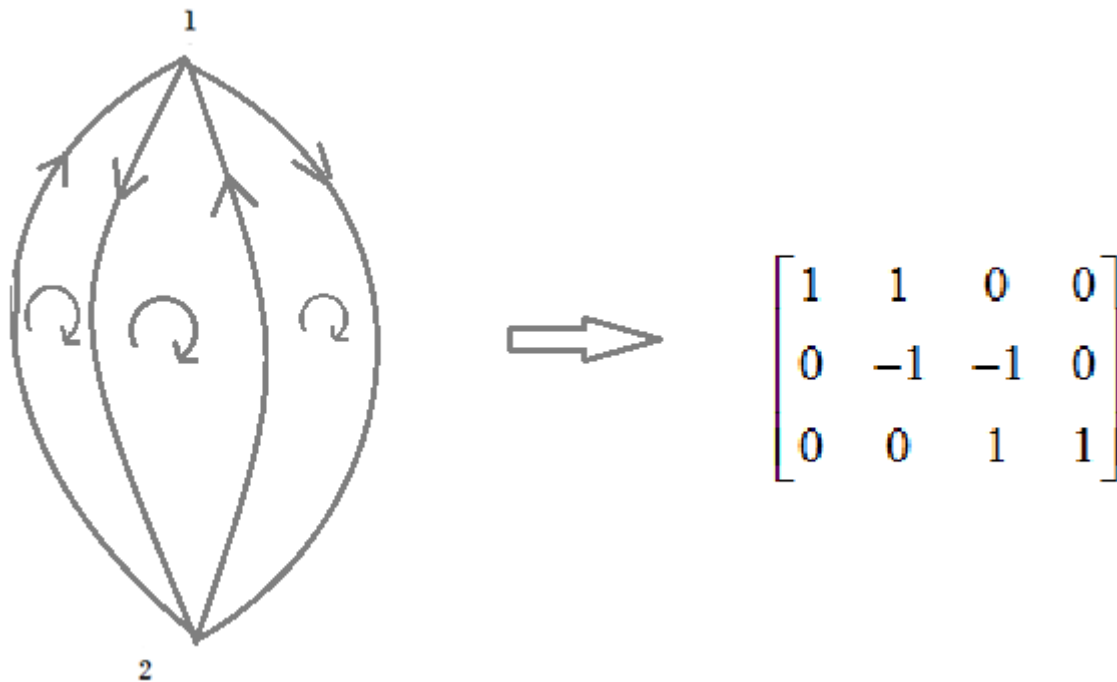


Figure:2. Graph with its Incidence Matrix

1.6.2. **Branch Impedance matrix $[Z_b]$:**

It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entries are zero.

III. REVIEW OF EXISTING LITERATURE

Network analysis is important branch in electrical engineering. It was successfully studied [3] from nineteenth century by German scientist Hermann von Helmholtz in 1853 and by Leon Charles Thevenin (1857–1926) an electrical engineer in 1883. Hans Ferdinand Mayer (1895–1980) in 1926 and Bell Labs engineer Edward Lawry Norton (1898–1983) also studied [3] about the electrical networks. There are various network theorems such as Thevenin’s theorem, Norton’s Theorem, Millman’s theorem, Superposition theorem, maximum power transfer theorem [3] etc. We studied all these theorems that these and it was observed that network theorems are little bit laborious and followed iterative method.

Therefore, we try to analyze the electrical networks with the help of network equilibrium equation of graph theory and try to overcome the limitations of the theorems with the following objectives:

IV. OBJECTIVES OF STUDY

During literature survey it was found that there are many limitations in the existing theorems in network analysis. Therefore, we proposed to analyze the electrical networks with the help of network equilibrium equation of graph theory and overcome the limitations of the theorems with the following objectives such as:

- (i) Study of electrical networks in details [3],[23],[7],[21].
- (ii) Study of graph theory [9],[18] in details.
- (iii) Establish the relation between graphs and networks.
- (iv) Find out the limitations of the existing theorems in network analysis.
- (v) Develop graph theoretic model to reduce the limitations of the theorems.
- (vi) Formulation of computer programming for graph theoretic model.

V. METHODOLOGY

Following the objectives first we have studied the laws of electrical circuits [18] and network theorems. Then we studied the basic rules of graph theory which relates the electrical circuits to matrices and graphs. Finally we successfully applied the network equilibrium equation of graph theory in network analysis to fulfil our objectives.

VI. NORTON'S THEOREM

Any electrical network containing any numbers of resistor and e.m.f. sources connected in any manner be enclosed in a box with two terminals A and B brought out.

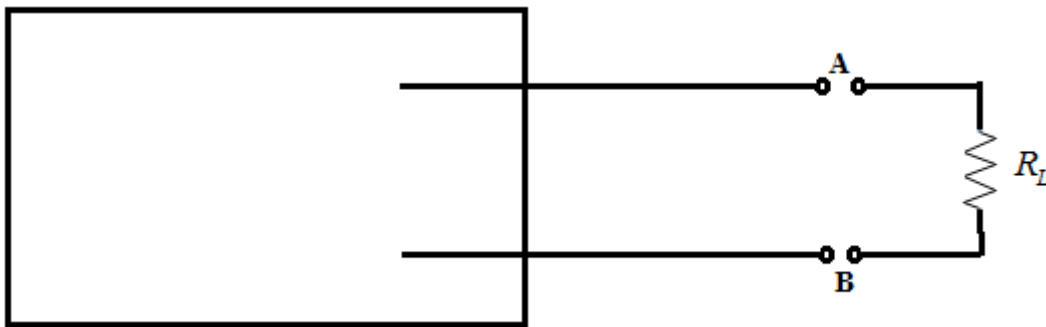


Figure 3: Electrical network with two terminals A and B

According to Norton's theorem the entire network behind AB can be replaced by a current source I_N in parallel with resistance R_N as shown in the figure.

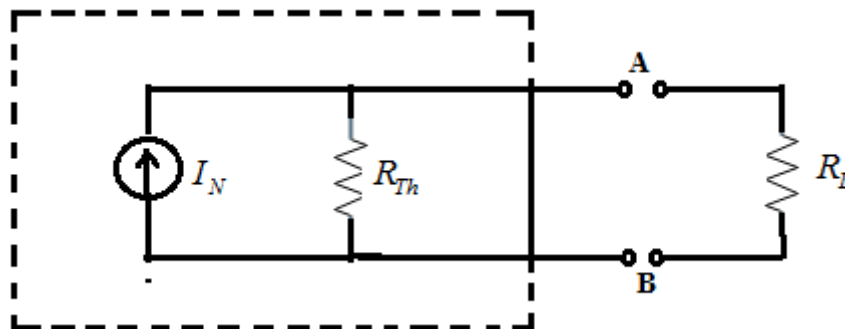


Figure 4: Norton's equivalent circuit

The Norton's theorem as applied to d.c. circuits may be stated as under:

Any linear, bilateral network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N .

- (i) The output I_N of the current source is equal to the current that would flow through AB when A and B are short-circuited.
- (ii) The resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.

Norton's Theorem is converse of Thevenin's theorem in that Norton equivalent circuit uses a current generator instead of voltage generator and the resistance R_N (which is the same as R_{Th}) in parallel with the generator instead of being in series with it. Thus the use of either of these theorems enables us to replace the entire circuit seen at a pair of terminals by an equivalent circuit made up of a single source and a single resistor.

VII. FINDING CURRENT USING NORTON'S THEOREM

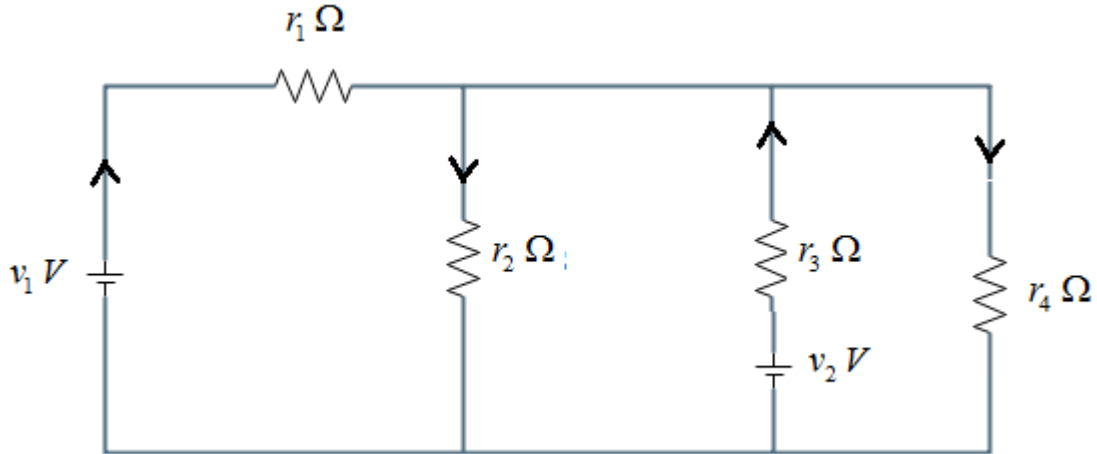


Figure 5: Circuit having arbitrary voltages and resistances

To find the Norton's equivalent circuit of the given circuit we first consider $v_1 V$ as Source

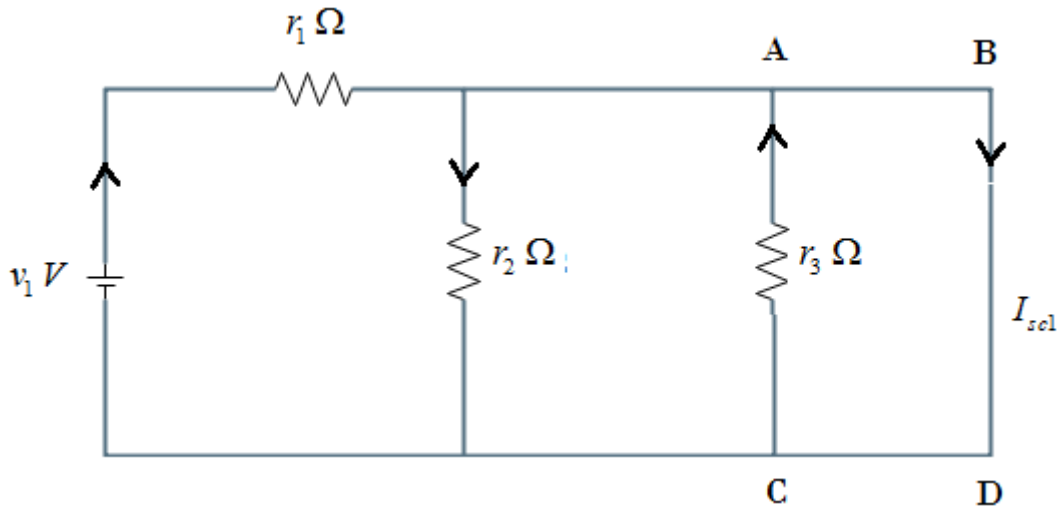


Figure 6: Circuit with $v_1 V$ only

Here resistance at $A-B-C-D=0$, as at point B resistance is zero due to short circuit and node A is parallel to node B.

So the circuit reduces to

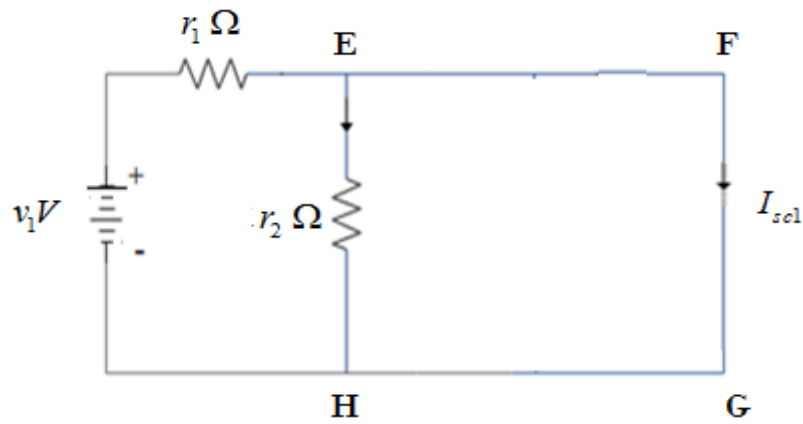


Figure 7: Reduced Circuit

Using the same logic at node E and F the circuit further reduces to

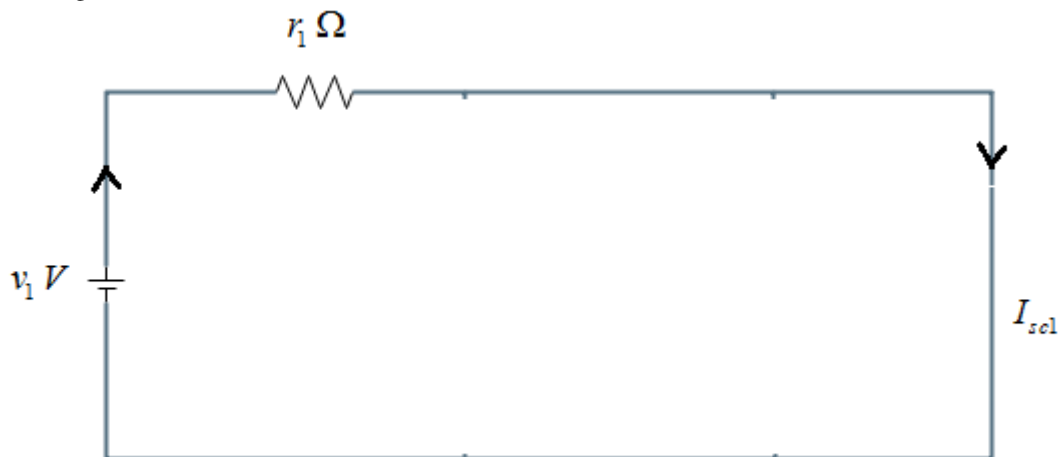


Figure 8: Reduced Circuit

Hence
$$I_{sc1} = \frac{v_1}{r_1} A$$

Now considering v_2V as source

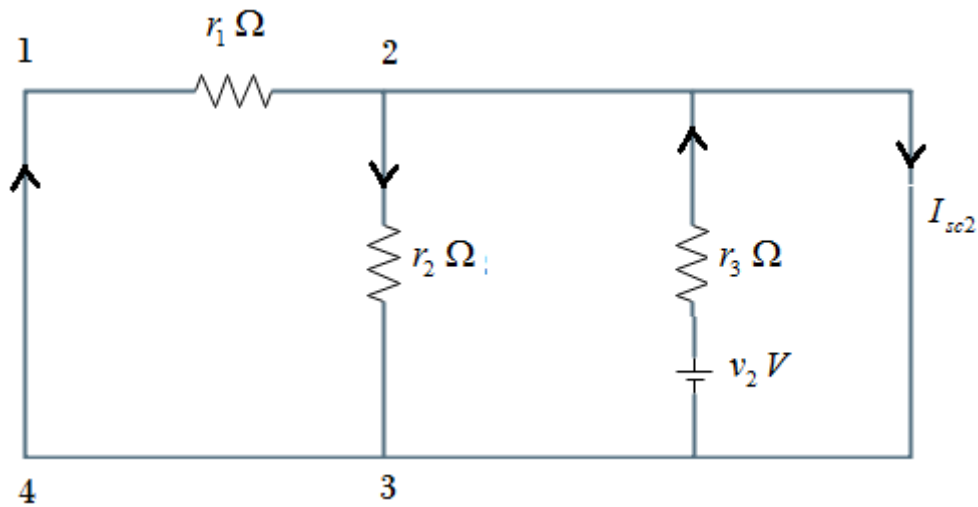


Figure 9: Circuit with v_2V

Here by using rules of parallel resistance of electrical circuit the circuit can be reduced to

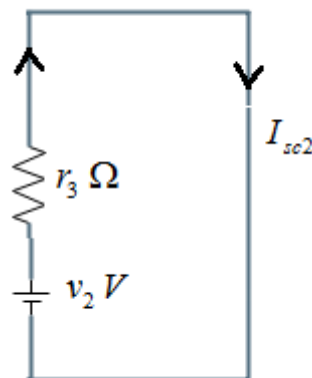


Figure 10: Reduced Circuit with v_2V

$$\text{So } I_{sc2} = \frac{v_2}{r_3} A$$

Thus Norton's equivalent current

$$I_N = I_{s.c1} + I_{s.c2} = \left(\frac{v_1}{r_1} + \frac{v_2}{r_3} \right) A$$

$$= \frac{r_3 v_1 + r_1 v_2}{r_1 r_3} A$$

Now to find the internal resistance R_{int} of the circuit neglecting the $r_4 \Omega$ resistance we have

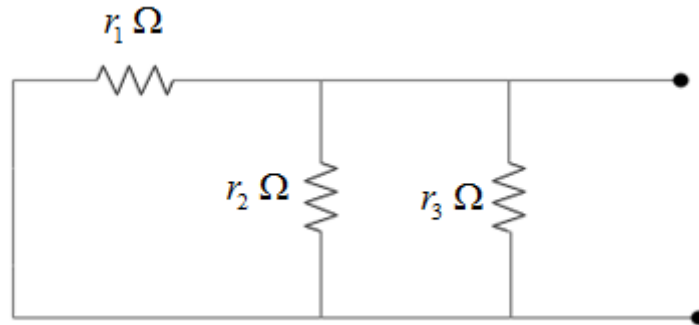


Figure 11: Circuit with internal resistance

$$\frac{1}{R_{\text{int}}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{r_1 r_2 + r_2 r_3 + r_3 r_1}{r_1 r_2 r_3}$$

$$\therefore R_{\text{int}} = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} \Omega$$

Using these Norton's equivalent circuit becomes

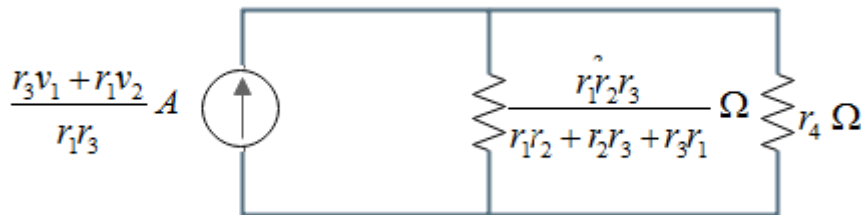


Figure 12: Norton's equivalent circuit

So using Norton's Theorem current through $r_4 \Omega$ resistance (say $I_{r_4 \Omega}$)

$$I_{r_4 \Omega} = I_N \frac{R_{\text{int}}}{R_{\text{int}} + r_4}$$

$$= \frac{r_3 v_1 + r_1 v_2}{r_1 r_3} \cdot \frac{\frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1}}{\frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} + r_4}$$

$$= \frac{r_2 r_3 v_1 + r_1 r_2 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2}$$

Graph theoretic approach

We will find the current through $r_4 \Omega$ resistor.

The graph from the circuit is given by

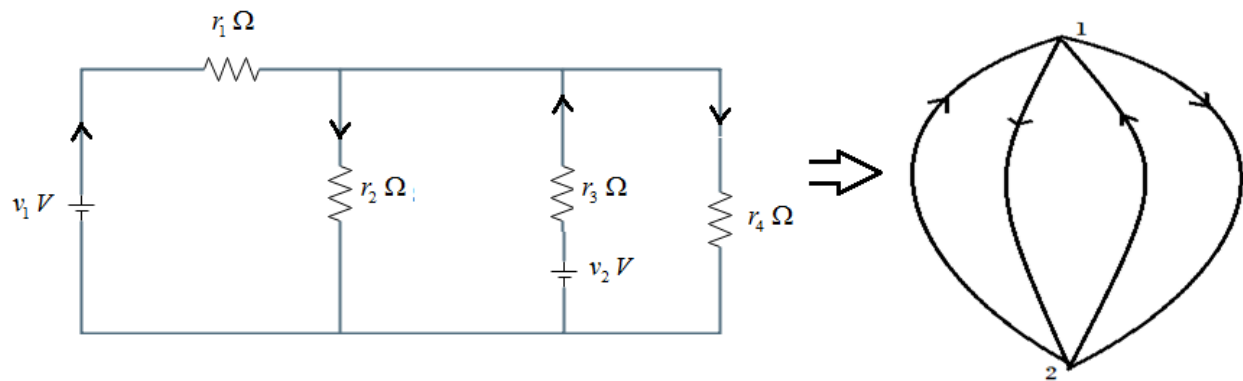


Figure 13: Graph from Circuit

Here

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, z_b = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, I_L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}, V_S = \begin{bmatrix} v_1 \\ 0 \\ v_2 \\ 0 \end{bmatrix}$$

So putting in

$$[B][Z_b][B^T][I_L] = -[B][V_S]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ 0 \\ v_2 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} r_1 + r_2 & -r_2 & 0 \\ -r_2 & r_2 + r_3 & -r_3 \\ 0 & -r_3 & r_3 + r_4 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} -v_1 \\ v_2 \\ -v_2 \end{bmatrix}$$

$$L_1 = - \frac{r_2 r_3 v_1 + r_2 r_4 v_1 + r_3 r_4 v_1 - r_2 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2}$$

$$L_2 = - \frac{r_2 r_3 v_1 + r_2 r_4 v_1 - r_1 r_4 v_2 - r_2 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2}$$

$$L_3 = - \frac{r_2 r_3 v_1 + r_1 r_2 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2} \quad \text{where } r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2 \neq 0$$

Also from

$$[I_b] = [B^T][I_L]$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{r_2 r_3 v_1 + r_2 r_4 v_1 + r_3 r_4 v_1 - r_2 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2} \\ -\frac{r_2 r_3 v_1 + r_2 r_4 v_1 - r_1 r_4 v_2 - r_2 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2} \\ -\frac{r_2 r_3 v_1 + r_1 r_2 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2} \end{bmatrix}$$

$$i_1 = -\frac{r_2 r_3 v_1 + r_2 r_4 v_1 + r_3 r_4 v_1 + r_2 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2},$$

$$i_2 = -\frac{r_3 r_4 v_1 + r_1 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2}$$

$$i_3 = \frac{r_2 r_4 v_1 - r_1 r_2 v_2 - r_1 r_4 v_2 - r_2 r_4 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2}$$

$$i_4 = -\frac{r_2 r_3 v_1 + r_1 r_2 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2}$$

Here the current through $r_4 \Omega$ resistor $i_4 = -\frac{r_2 r_3 v_1 + r_1 r_2 v_2}{r_1 r_2 r_3 + r_2 r_3 r_4 + r_3 r_4 r_1 + r_4 r_1 r_2} A$

In this case also we see that the current obtained by Norton's theorem is same as the current obtained by graph theoretic approach.

From our study it is observed that as both the method gives the same current. And also observed that the graph theoretic method is much more less time consuming and easier than the Norton's theorem.

VIII. CONCLUSION

In view of above from our study it is clear that graph theoretic approach is the best effective method for network analysis or alternatively we can conclude that Norton's theorems can be replaced by graph theoretic model. In further study one can study the other electrical theorems using this graph theoretic model.

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